

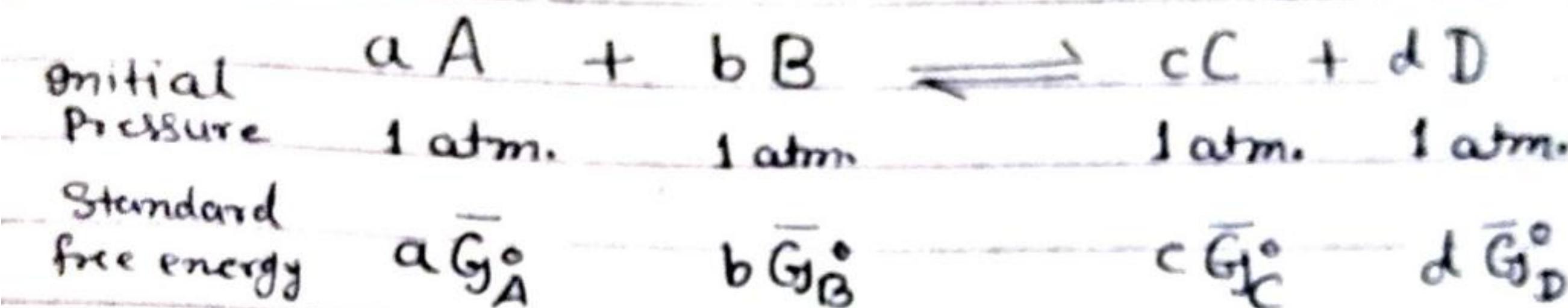
T.D.C Part II : Chem. (Hon.)

Paper IV ; Gr. A (Phys. Chem.)
Unit II Chemical Equilibrium

Vant-Hoff reaction Isotherm

The equation gives the increase of free energy for the transfer of reactant at any concentration or partial pressure to products.

For the following reaction in the gas state, if all the reactants and the products be in the standard state i.e. at one atm. pressure and const. temp. T.



∴ Standard free energy change,

$$\Delta G^\circ = c\bar{G}_C^\circ + d\bar{G}_D^\circ - a\bar{G}_A^\circ - b\bar{G}_B^\circ$$

if at any specified state, let

$$\text{Partial pressure} = p_A \quad p_B \quad p_C \quad p_D$$

$$\text{Free energy} = a\bar{G}_A - b\bar{G}_B - c\bar{G}_C - d\bar{G}_D$$

∴ Free energy, (ΔG)

$$\Delta G = cG_C + dG_D - aG_A - bG_B$$

For an infinitesimal change in free energy and partial pressure of any component,

$$dG = vdp - SdT$$

$$= vdp \quad (\text{at const. } T)$$

$$= nRT \frac{dp}{p} \quad (\text{for } n \text{ moles of an ideal gas})$$

Thus, for A,

$$dG = \alpha RT d\ln p$$

For A, when pressure changes from 1 to p_A

Free energy changes from

$$\alpha \bar{G}_A^\circ \rightarrow \alpha \bar{G}_A$$

Integrating between the above limits.

$$\int_{\alpha \bar{G}_A^\circ}^{\alpha \bar{G}_A} dG = \alpha RT \int_1^{p_A} d\ln p$$

$$\text{or, } \alpha \bar{G}_A - \alpha \bar{G}_A^\circ = \alpha RT \ln p_A / 1$$

Similarly,

$$b \bar{G}_B - b \bar{G}_B^\circ = b RT \ln p_B$$

$$c \bar{G}_C - c \bar{G}_C^\circ = c RT \ln p_C$$

$$d \bar{G}_D - d \bar{G}_D^\circ = d RT \ln p_D$$

Subtracting ΔG° from ΔG

$$\begin{aligned} \Delta G - \Delta G^\circ &= (c \bar{G}_C - c \bar{G}_C^\circ) + (d \bar{G}_D - d \bar{G}_D^\circ) \\ &\quad - (\alpha \bar{G}_A - \alpha \bar{G}_A^\circ) - (b \bar{G}_B - b \bar{G}_B^\circ) \end{aligned}$$

$$= c RT \ln p_C + d RT \ln p_D$$

$$- \alpha RT \ln p_A - b RT \ln p_B$$

$$= RT [c \ln p_C + d \ln p_D - \alpha \ln p_A - b \ln p_B]$$

$$\text{or, } \Delta G_J = RT \ln \frac{\frac{P_c^c}{P_A^a} \cdot \frac{P_D^d}{P_B^b}}{} + \Delta G^\circ \quad (1)$$

$$= \Delta G^\circ + RT \sum v \ln p.$$

Where, \bar{G}_J = partial molar free energy.
 v = no. of molecules of space.

The above equation is known as
 Vant-Hoff reaction isotherm.

Uses of reaction isotherm :→

1. Equilibrium constant of a reaction :→

At equilibrium at const. T & P ,

Since,

$$\begin{aligned} dG_J &= vdp - SdT \\ &= 0 \quad (dp=0, dt=0) \end{aligned}$$

$\therefore \Delta G_J = 0$ for a finite change

Hence,

Vant-Hoff reaction isotherm

$$\Delta G_J = \Delta G^\circ + RT \sum v \ln p$$

becomes,

$$0 = \Delta G^\circ + RT \sum v \ln p$$

$$\text{or, } -\Delta G^\circ = +RT \sum v \ln p$$

$$\begin{aligned} &= RT(c \ln p_c + d \ln p_D \\ &\quad - a \ln p_A - b \ln p_B) \end{aligned}$$

$$\text{or} \quad -\Delta G_j^\circ = RT \ln \frac{p_c^c \cdot p_d^d}{p_a^a \cdot p_b^b}$$

ΔG_j° is a constant as it corresponds to Standard state and therefore its $T = \text{const.}$

$$\frac{p_c^c \cdot p_d^d}{p_a^a \cdot p_b^b} = \text{const.} = K_p \text{ is constant called equilibrium const.}$$

K_p is a constant at constant T .

② ΔG_j° from equilibrium constant \rightarrow

At equilibrium ~~constant~~ at constant T & P .

$$\Delta G_j = 0$$

So that, by V-H reaction isotherm

$$\Delta G_j = \Delta G_j^\circ + RT \sum v \ln p$$

here here,

$$-\Delta G_j^\circ = RT \sum v \ln p$$

$$= RT \ln p \quad (\text{at equilibrium})$$

$$= -2.303 \log_{10} p$$

$$= 2.303 \log_{10} K_p$$

③ $\Delta G_j \rightarrow$

By V-H reaction isotherm

$$\Delta G_j = \Delta G_j^\circ + RT \sum v \ln p$$

$$= -RT \ln K_p + RT \sum v \ln p$$

Having determined, ΔG_j can be calculated at partial pressure p_A — of the reactants and products of the reaction.

Thermodynamic Equilibrium constant \rightarrow

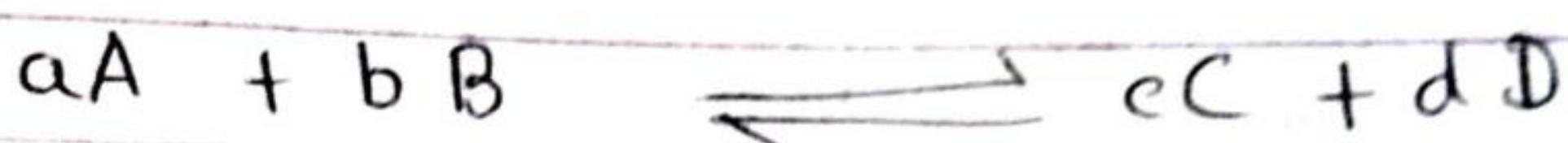
If activity 'a' is used for concentration.

$$K_a = \frac{a^c \cdot a^d}{a^a \cdot a^b}$$

Where, K_a is called thermodynamic equilibrium constant.

Relation between K_a , K_c and $K_f \rightarrow$

For the reaction,



Equilibrium constant

$$K_a = \frac{C_c^c \cdot C_d^d}{C_A^a \cdot C_B^b}$$

As, $a = f_a C$.

Where, f = activity co-efficient

a = activity.

$$K_c = \frac{(a_c/f_c)^c \cdot (a_d/f_d)^d}{(a_A/f_a)^a \cdot (a_B/f_B)^b}$$

$$= \frac{a_c^c \cdot a_d^d}{a_A^a \cdot a_B^b} \cdot \frac{f_A^a \cdot f_B^b}{f_c^c \cdot f_d^d}$$

$$= K_a \times \frac{f_A^a \cdot f_B^b}{f_c^c \cdot f_d^d}$$

$$= K_a \quad (\text{in infinite dilute solution} \\ f=1)$$

In a gas reaction,

activity, 'a' = fugacity 'f'

gf γ = activity co-efficient

$$f = a = \beta \gamma$$

where, β = partial pressure.

$$\therefore K_f = \frac{f_c^c \cdot f_d^d}{f_A^a \cdot f_B^b}$$

Equilibrium constant using fugacity.

$$= \frac{p_c^c \gamma_c^c \cdot p_d^d \gamma_d^d}{p_A^a \gamma_A^a \cdot p_B^b \gamma_B^b}$$

$$= \frac{p_c^c \cdot p_d^d}{p_A^a \cdot p_B^b} \cdot \frac{\gamma_c^c \cdot \gamma_d^d}{\gamma_A^a \cdot \gamma_B^b}$$

$$= K_p \cdot \left(\frac{\gamma_c^c \cdot \gamma_d^d}{\gamma_A^a \cdot \gamma_B^b} \right)$$